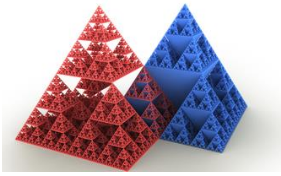


ECE 6564 – Nonlinear Dynamics and Chaos

Spring 2025

Georgia Tech Europe

Brief Description

Introduction to the nonlinear dynamics of continuous-time and discrete-time systems. Routes to chaos. Quantification of chaos. Nonlinear time series analysis. Applications of Chaos Theory in Electrical and Computer Engineering.
TIA: Systems and Controls.

Textbook

K.T. Alligood, T.D. Sauer, and J.A. Yorke, *Chaos. An introduction to dynamical systems,* Springer-Verlag, 1996. ISBN 0-387-94677-2 (required).

Interesting additional books (optional)
E. Ott, *Chaos in Dynamical Systems,* Second Edition, Cambridge University Press, 2002. ISBN 978-0-521-01084-9 (optional).
S.H. Strogatz, *Nonlinear Dynamics and Chaos with Applications to Physics, Biology, Chemistry, and Engineering*, Second Edition, Cambridge University Press, 2004. ISBN 978-0521529020 (optional).
H. Kantz and T. Schreiber, *Nonlinear Time Series Analysis,* Second Edition, Westview Press, 2015. ISBN 978-0-8133-4910-7 (optional).

Prerequisites

ECE 6550 or equivalent

Instructor

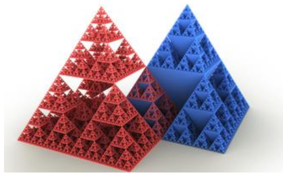
Dr. Alexandre Locquet, Office 206.
Communication: Please send me messages using Canvas. ("Inbox" tab on your dashboard)

Lectures

TBD

Office hours

TBD



Assessment

Homework	**15%**
Project	**55%**
Final Exam	**30%**
In-lecture polling	**2% (bonus)**
Attendance	**1% (bonus)**

Important Dates

Project Presentation 1	**TBD**
Project Presentation 2	**TBD**
Project Presentation 3	**TBD**
Final Report due	**TBD**
Final Exam	**TBD**

Homework

Around 4 problem sets will be assigned. An assignment might be due on the week before final exams. Please box all answers. Homework should be submitted electronically on Canvas as a single, legible, pdf file. You are allowed to collaborate on homework with other students, but all work to be submitted should then be worked out and written up on your own. Of course, copying solutions from a Faculty solutions manual is cheating.
A completion grade will be assigned. If a given homework 1) has been submitted on time and 2) every problem is answered, the student gets 3.75% credit.
Homework turned in late loses 50%. Homework turned in two or more days late will not earn credit.

Final Exam

The final examination will be 3-hour long and comprehensive.

Project

You will choose a project related to a specific subject in nonlinear dynamics at the beginning of the term. You are required to give a 5-minute presentation around February 1st, a 10-minute presentation around March 12, and a final 15-minute presentation around April 18. You also need to provide a 10-page final report.

Examples of possible topics:

- Chaos in electronic circuits
- Synchronization of chaotic electronic circuits
- Communications with chaotic electronic systems
- Chaos-based encryption
- Physical-layer security using chaotic systems, secure key distribution
- Chaos control
- Stability/instability of electric power grids
- Chaos in lasers and synchronization of chaotic lasers
- Pattern formation in spatially extended systems
- Stochastic Resonance
- Embedding theorems (nonlinear time series analysis) and their applications
- Reservoir (neuromorphic) computing with nonlinear dynamical systems
- Compartmental models in epidemiology (e.g. SIR, SIS, SEIR)
- Design of fuel efficient space missions (Lagrange points, Interplanetary Transport Network,…)
- Chaotic Mixing of microfluids
- Chaos synchronization of networks of dynamical systems (cluster formation, chimera states, etc…)
- Heart nonlinear dynamics/ Sudden death forecasting
- Nonlinear dynamics in neuron behavior /inter-neuron communication
- Rogue wave
- Nonlinear dynamics in gene regulation

Students will typically make extensive use of the digital library: http://www.library.gatech.edu/ to get started on the project.

Attendance

Institute Absence Policy can be found here: http://www.catalog.gatech.edu/rules/4/. In addition, a 1% BONUS is associated to attendance. Attendance Polling using PointSolutions Technology and/or attendance sheets will be used to count the number of absences.
Students need to install the PointSolutions app on a mobile device. As a Georgia Tech student, installation and use of the app is free. In the PointSolutions settings, the region needs to be set to "North/South America" and NOT to Europe.
Students who do not miss more than 2 lectures during the entire term get a 1% attendance BONUS. Students who miss more than 2 classes get 0% for the attendance BONUS.

In-Lecture Polling

You will be asked to answer questions during some lectures, using the PointSolutions app. Students who will have correctly answered at least 80% of all questions asked get a 2% bonus.

Topical Outline

1. Dynamics of Iterated Maps

One- and Two-dimensional maps
Limit sets: fixed points, periodic points, chaotic attractors
Stability: sinks, sources, saddles, stable and unstable manifolds.

Chaotic Orbits: sensitivity to initial conditions, Lyapunov exponents, fractals.

2. Dynamics of Continuous-Time Systems

Linear and nonlinear systems. Poincaré-Bendixson theorem.
Limit sets: Equilibrium points, limit cycles, quasi-periodic attractors, chaotic attractors.
Stability: characteristic values, Lyapunov functions, Floquet multipliers, Lyapunov exponents.
Lab demonstrations: the Chua circuit, the Lorenz circuit, chaos in laser diodes.

3. Bifurcations

Saddle-node, period-doubling, Hopf, and torus bifurcations.
Bifurcation diagrams and routes to chaos: period-doubling cascade, quasi-periodicity, intermittency and crises.
Lab demonstration: period-doubling cascade in the Lorenz system, bifurcations in the Chua circuit.

4. Quantifying chaos

Lyapunov spectrum.
Fractal dimensions.
Kolmogorov-Sinai Entropy.

5. Basics of nonlinear time series analysis

State reconstruction from data through delay embedding.
Simple nonlinear prediction and noise reduction.
Computation of the largest Lyapunov exponent from data

6. Coupled chaotic systems and applications in electrical and computer engineering

Chaos control.
Synchronization of chaotic systems and circuits.
Chaos-based telecommunications.
Optical communications using chaotic lasers
Random-number generation
Neuromorphic computing
Tackling NP-hard problems
Compressive Sensing

Course Outcomes

After successfully completing this course, students should be able to:

1. analyze the dynamical behavior of nonlinear dynamical systems described by nonlinear maps,
2. analyze the dynamical behavior of dynamical systems described by nonlinear ordinary differential equations,
3. simulate numerically in the time and phase-space domains the evolution of nonlinear dynamical systems,
4. plot and interpret bifurcation diagrams,

5. understand various measures of complexity of the steady-state behavior of dynamical systems and in particular the Lyapunov spectrum and attractor dimension,
6. utilize software to compute various complexity measures of dynamical systems,
7. analyze scientific literature related to dynamical systems, synthesize and present the information to an audience of non-specialists.

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I. Basic Notions on Dynamical systems
I.1 What is Chaos?
I.2 Classification of Dynamical Systems.
I.2.1 What is a Dynamical System?
I.2.2 Classification

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II.1 Introduction and Definitions
II.2 Stability of Fixed Points
II.3 Periodic Orbits
II.4 Sensitive Dependence on Initial Conditions
II.5 Itineraries and Symbolic Dynamics
II.6 Bifurcations of Smooth One-Dimensional Maps
II.6.1 Period-Doubling Bifurcation
II.6.2 Saddle-Node Bifurcation
II.6.3 The Transcritical Bifurcation

III. Two-Dimensional Maps
III.1 Introduction and Definitions
III.2 Stability of Fixed Points
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III.4.1 Stability of Fixed and Periodic Points
III.4.2 Stable and Unstable Manifolds of Saddle Points
III.5 (Local) Bifurcations in 2D Maps
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IV.3 Chaotic Orbits
IV.4 Chaos in the Logistic Map G=4x(1-x)
IV.4.1 Conjugacy
IV.5 Lyapunov Exponents in Higher Dimensions
IV.6 Chaotic Orbits in Higher Dimensions
IV.7 Numerical Calculation of Lyapunov Exponents
Iv.8 Chaotic Attractors
IV.8.1 Limit Sets
IV.8.2 Dimension of Chaotic Attractors

V. Continuous-Time Systems
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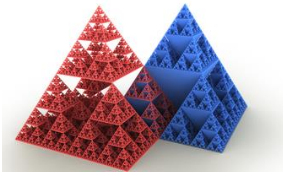
V.2 Existence and Uniqueness
V.3 Linear 1D Differential Equations
V.4 Equilibrium and Stability
V.5 Nonlinear 1D Differential Equations



V.6 n-Dimensional Linear Differential Equations
V.7 StabilityCriterion for Linear and Nonlinear Systems
V.8 Nonlinear Differential Equations in More than One Dimension
V.8.1 Properties of forward-limit sets
V.8.2 The Poincaré-Bendixson Theorem
V.9 Chaos in Differential Equations
V.9.1 Quasi-Periodic Signals
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V.9.3 Example of Chaotic Systems
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V.10.1 Saddle-Node Bifurcation
V.10.2 Andronov-Hopf Bifurcation
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V.11.1 Saddle-Node Bifurcations of Limit Cycles
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VI.3 Examples: modeling, prediction, parameter identification, estimation of attractor dimension

VII Applications of Chaos Theory in Electrical and Computer Engineering
VII.1 Chaos in Electronic Systems
VII.2 Chaos in Laser Systems
VII.3 Synchronization of Chaotic Systems
VII.4 Communications with Chaotic Systems and CircuitsVII.5 Chaos Control
VII.6 Random-Number Generation
VII.7 Neuromorphic Computing and NP-hard problems
VII.8 Compressive sensing exploiting the pseudo-randomness of a chaotic system

Student-Faculty Expectations Agreement

At Georgia Tech we believe that it is important to strive for an atmosphere of mutual respect, acknowledgement, and responsibility between faculty members and the student body. Please refer to Student-Faculty Expectations for an articulation of some basic expectation that you can have of me and that I have of you. In the end, simple respect for knowledge, hard work, and cordial interactions will help build the environment we seek. Therefore, I encourage you to remain committed to the ideals of Georgia Tech while in this class.

Honor Code

Students are, of course, expected to abide by the *Academic Honor Code* and the "GTL student policies" document that you will find in the resources section of the Canvas website for this course. Instances of academic misconduct will be viewed very seriously. For any questions involving

Academic Honor Code issues, please consult me or visit http://www.policylibrary.gatech.edu/student-affairs/academic-honor-code.

Disability Services

The Office of Disability Services collaborates with students, faculty, and staff to create a campus environment that is usable, equitable, sustainable, and inclusive of all members of the Georgia Tech community. Disability is an aspect of diversity integral to society and Georgia Tech. If students encounter academic, physical, technological, or other barriers on campus, the Disability Services team collaborates with the student to find creative solutions and reasonable accommodation. For any question regarding Disability Services, please consult Mrs. Corinne Guyot or visit https://disabilityservices.gatech.edu/.

Student Feedback

Anonymous feedback can be provided to the instructor using the link below:

https://docs.google.com/forms/d/e/1FAIpQLSdPBLPAEyohxyQY-w6nBh-dznH4sPP52Mx6XZKwI3wLBS6obg/viewform?usp=sf_link

You are also encouraged to fill in the course-instructor opinion survey (CIOS).